29. (II) Determine the magnitudes and directions of the currents in each resistor shown in Fig. 19–48. The batteries have emfs of $\mathscr{E}_1 = 9.0$ V and $\mathscr{E}_2 = 12.0$ V and the resistors have values of $R_1 = 25\ \Omega$, $R_2 = 18\ \Omega$, and $R_3 = 35\ \Omega$.

$I_1 + I_2 = I_3$

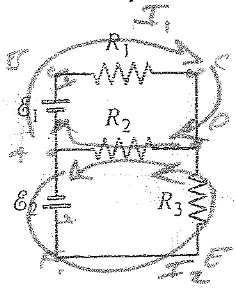


FIGURE 19–48 Problems 29 and 30.

30. (II) Repeat Problem 29, assuming each battery has internal resistance $r = 1.0\ \Omega$.

31. (II) Calculate the currents in each resistor of Fig. 19–49.

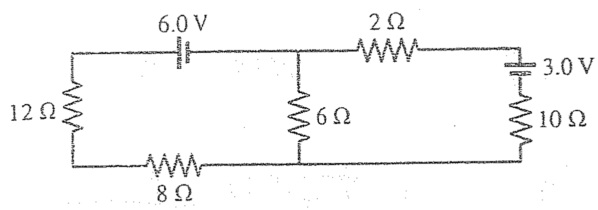
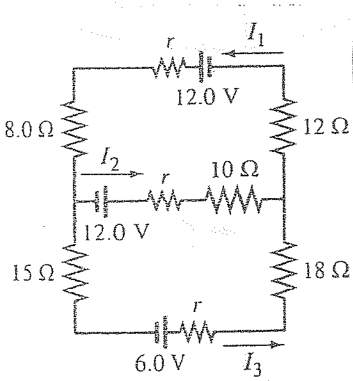


FIGURE 19–49 Problem 31.

32. (III) (*a*) Determine the currents I_1, I_2, and I_3 in Fig. 19–50. Assume the internal resistance of each battery is $r = 1.0\ \Omega$. (*b*) What is the terminal voltage of the 6.0-V battery?



$I_1 = 0.77$ A
$I_2 = 0.71$ A
$I_3 = 0.055$ A

FIGURE 19–50 Problems 32 and 33.

33. (III) What would the current I_1 be in Fig. 19–50 if the 12-Ω resistor is shorted out? Let $r = 1.0\ \Omega$.

19–4 Emfs Combined, Battery Charging

*34. (II) Suppose two batteries, with unequal emfs of 2.00 V and 3.00 V, are connected as shown in Fig. 19–51. If each internal resistance is $r = 0.100\ \Omega$, and $R = 4.00\ \Omega$, what is the voltage across the resistor R?

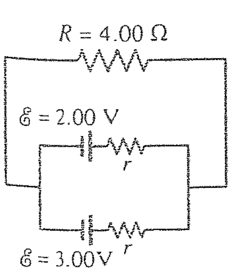


FIGURE 19–51 Problem 34.

24. (II) Determine the terminal voltage of each battery in Fig. 19–44.

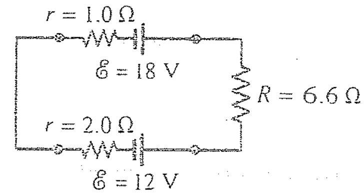


FIGURE 19–44 Problem 24.

25. (II) (*a*) What is the potential difference between points a and d in Fig. 19–45 (same circuit as Fig. 19–13, Example 19–8), and (*b*) what is the terminal voltage of each battery?

a) −25.7 V
b) 77.4 V
43.3 V

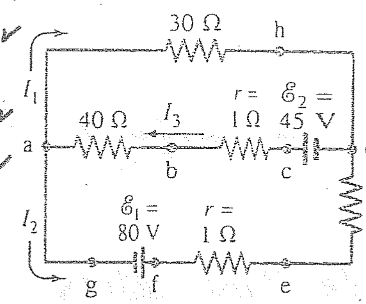


FIGURE 19–45 Problem 25.

26. (II) For the circuit shown in Fig. 19–46, find the potential difference between points a and b. Each resistor has $R = 75\ \Omega$ and each battery is 1.5 V.

0 V

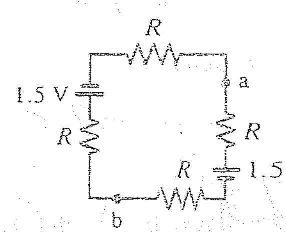


FIGURE 19–46 Problem 26.

27. (II) Determine the magnitudes and directions of the currents through R_1 and R_2 in Fig. 19–47.

0.68 A ←
0.40 A ←

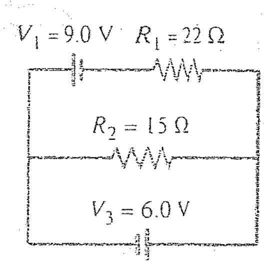


FIGURE 19–47 Problems 27 and 28.

28. (II) Repeat Problem 27, now assuming that each battery has an internal resistance $r = 1.2\ \Omega$.

24. Apply Kirchhoff's loop rule to the circuit starting at the upper left corner of the circuit diagram, in order to calculate the current. Assume that the current is flowing clockwise.

$$-I(1.0\Omega)+18\text{ V}-I(6.6\ \Omega)-12\text{ V}-I(2.0\Omega)=0 \ \rightarrow\ I=\frac{6\text{ V}}{9.6\ \Omega}=0.625\text{A}$$

The terminal voltage for each battery is found by summing the potential differences across the internal resistance and EMF from left to right. Note that for the 12 V battery, there is a voltage gain going across the internal resistance from left to right.

$$18\text{ V battery: } V_{\text{terminal}}=-I(1.0\Omega)+18\text{ V}=-(0.625\text{ A})(1.0\Omega)+18\text{ V}=\boxed{17.4\text{ V}}$$

$$12\text{ V battery: } V_{\text{terminal}}=I(2.0\Omega)+12\text{ V}=(0.625\text{ A})(2.0\Omega)+12\text{ V}=\boxed{13.3\text{ V}}$$

25. From Example 19-8, we have $I_1=-0.87\text{ A}$, $I_2=2.6\text{ A}$, $I_3=1.7\text{ A}$. If another significant figure had been kept, the values would be $I_1=-0.858\text{ A}$, $I_2=2.58\text{ A}$, $I_3=1.73\text{ A}$. We use those results.

(*a*) To find the potential difference between points a and d, start at point a and add each individual potential difference until reaching point d. The simplest way to do this is along the top branch.

$$V_{\text{ad}}=V_{\text{d}}-V_{\text{a}}=-I_1(30\Omega)=-(0.858\text{ A})(30\Omega)=\boxed{-25.7\text{ V}}$$

Slight differences will be obtained in the final answer depending on the branch used, due to rounding. For example, using the bottom branch, we get the following.

$$V_{\text{ad}}=V_{\text{d}}-V_{\text{a}}=\mathcal{E}_1-I_2(21\Omega)=80\text{ V}-(2.58\text{ A})(21\Omega)=-25.8\text{ V}$$

(*b*) For the 80-V battery, the terminal voltage is the potential difference from point g to point e. For the 45-V battery, the terminal voltage is the potential difference from point d to point b.

$$80\text{ V battery: } V_{\text{terminal}}=\mathcal{E}_1-I_2r=80\text{ V}-(2.58\text{ A})(1.0\Omega)=\boxed{77.4\text{ V}}$$

$$45\text{ V battery: } V_{\text{terminal}}=E_2-I_3r=45\text{ V}-(1.73\text{ A})(1.0\Omega)=\boxed{43.3\text{ V}}$$

26. To find the potential difference between points a and b, the current must be found from Kirchhoff's loop law. Start at point a and go counterclockwise around the entire circuit, taking the current to be counterclockwise.

$$-IR+\mathcal{E}-IR-IR+\mathcal{E}-IR=0 \ \rightarrow\ I=\frac{\mathcal{E}}{2R}$$

$$V_{\text{ab}}=V_{\text{a}}-V_{\text{b}}=-IR+\mathcal{E}-IR=\mathcal{E}-2IR=\mathcal{E}-2\frac{\mathcal{E}}{2R}R=\boxed{0\text{ V}}$$

27. Because there are no resistors in the bottom branch, it is possible to write Kirchhoff loop equations that only have one current term, making them easier to solve. To find the current through R_1, go around the outer loop counterclockwise, starting at the lower left corner.

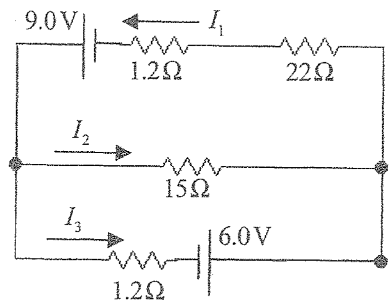
$$V_3-I_1R_1+V_1=0 \ \rightarrow\ I_1=\frac{V_3+V_1}{R_1}=\frac{6.0\text{ V}+9.0\text{ V}}{22\Omega}=\boxed{0.68\text{ A, left}}$$

To find the current through R_2, go around the lower loop counterclockwise, starting at the lower left corner.

$$V_3-I_2R_2=0 \ \rightarrow\ I_2=\frac{V_3}{R_2}=\frac{6.0\text{ V}}{15\Omega}=\boxed{0.40\text{ A, left}}$$

28. There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches on the left of the circuit.

$$I_1 = I_2 + I_3$$



Another equation comes from Kirchhoff's loop rule applied to the outer loop, starting at the lower left corner, and progressing counterclockwise.

$$-I_3(1.2\,\Omega)+6.0\,\text{V}-I_1(22\,\Omega)-I_1(1.2\,\Omega)+9.0\,\text{V}=0 \rightarrow$$
$$15 = 23.2I_1 + 1.2I_3$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the lower left corner, and progressing counterclockwise.

$$-I_3(1.2\,\Omega)+6.0\,\text{V}+I_2(15\,\Omega)=0 \rightarrow 6=-15I_2+1.2I_3$$

Substitute $I_1 = I_2 + I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$15 = 23.2I_1 + 1.2I_3 = 23.2(I_2+I_3)+1.2I_3 = 23.2I_2 + 24.4I_3 \; ; \; 6 = -15I_2 + 1.2I_3$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$6 = -15I_2 + 1.2I_3 \rightarrow I_2 = \frac{-6+1.2I_3}{15}$$

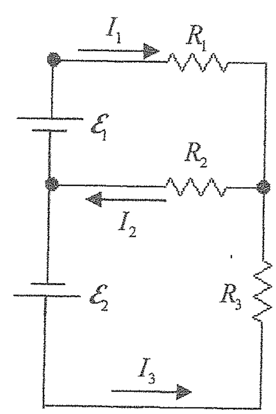
$$15 = 23.2I_2 + 24.4I_3 = 23.2\left(\frac{-6+1.2I_3}{15}\right)+24.4I_3 \rightarrow 225 = -138 + 27.84I_3 + 366I_3 \rightarrow$$

$$I_3 = \frac{363}{393.84} = 0.9217\,\text{A} \; ; \; I_2 = \frac{-6+1.2I_3}{15} = \frac{-6+1.2(0.9217)}{15} = -0.3263\,\text{A} \approx \boxed{0.33\,\text{A, left}}$$

$$I_1 = I_2 + I_3 = 0.5954\,\text{A} \approx \boxed{0.60\,\text{A, left}}$$

29. There are three currents involved, and so there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction of the three branches on the right of the circuit.

$$I_2 = I_1 + I_3 \rightarrow I_1 = I_2 - I_3$$



Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the battery and progressing clockwise.

$$E_1 - I_1R_1 - I_2R_2 = 0 \rightarrow 9 = 25I_1 + 18I_2$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$E_2 - I_3R_3 - I_2R_2 = 0 \rightarrow 12 = 35I_3 + 18I_2$$

Substitute $I_1 = I_2 - I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$9 = 25I_1 + 18I_2 = 25(I_2 - I_3) + 18I_2 = 43I_2 - 25I_3 \; ; \; 12 = 35I_3 + 18I_2$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$6.0\text{ V}-I_1(12\Omega)-I_1(8\Omega)-I_2(6\Omega)=0 \rightarrow 6=20I_1+6I_2 \rightarrow 3=10I_1+3I_2$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the battery and progressing counterclockwise.

$$3.0\text{ V}-I_3(2\Omega)+I_2(6.0\Omega)-I_3(10\Omega)=0 \rightarrow 3=-6I_2+12I_3 \rightarrow 1=-2I_2+4I_3$$

Substitute $I_1=I_2+I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$3=10I_1+3I_2=10(I_2+I_3)+3I_2=13I_2+10I_3 \ ; \ 1=-2I_2+4I_3$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$1=-2I_2+4I_3 \rightarrow I_2=\frac{4I_3-1}{2}$$

$$3=13I_2+10I_3=13\left(\frac{4I_3-1}{2}\right)+10I_3 \rightarrow 6=52I_3-13+20I_3 \rightarrow$$

$$I_3=\frac{19}{72}=0.2639\text{ A}\approx 0.26\text{ A} \ ; \ I_2=\frac{4I_3-1}{2}=0.0278\text{ A}\approx 0.028\text{ A}$$

$$I_1=I_2+I_3=0.2917\text{ A}\approx 0.29\text{ A}$$

The current in each resistor is as follows:

$$\boxed{2\Omega: 0.26\text{ A} \quad 6\Omega: 0.028\text{ A} \quad 8\Omega: 0.29\text{ A} \quad 10\Omega: 0.26\text{ A} \quad 12\Omega: 0.29\text{ A}}$$

32. Since there are three currents to determine, there must be three independent equations to determine those three currents. One comes from Kirchhoff's junction rule applied to the junction near the negative terminal of the middle battery.

$$I_1=I_2+I_3$$

Another equation comes from Kirchhoff's loop rule applied to the top loop, starting at the negative terminal of the middle battery, and progressing counterclockwise.

$$12.0\text{ V}-I_2(1.0\Omega)-I_2(10\Omega)-I_1(12\Omega)+12.0\text{ V}-I_1(1.0\Omega)-I_1(8.0\Omega)=0 \rightarrow$$

$$24=11I_2+21I_1=0$$

The final equation comes from Kirchhoff's loop rule applied to the bottom loop, starting at the negative terminal of the middle battery, and progressing clockwise.

$$12.0\text{ V}-I_2(1.0\Omega)-I_2(10\Omega)+I_3(18\Omega)+I_3(1.0\Omega)-6.0\text{ V}+I_3(15\Omega)=0 \rightarrow$$

$$6=11I_2-34I_3$$

Substitute $I_1=I_2+I_3$ into the top loop equation, so that there are two equations with two unknowns.

$$24=11I_2+21I_1=11I_2+21(I_2+I_3)=32I_2+21I_3 \ ; \ 6=11I_2-34I_3$$

Solve the bottom loop equation for I_2 and substitute into the top loop equation, resulting in an equation with only one unknown, which can be solved.

$$6=11I_2-34I_3 \rightarrow I_2=\frac{6+34I_3}{11}$$

$$24=32I_2+21I_3=32\left(\frac{6+34I_3}{11}\right)+21I_3 \rightarrow 264=192+1088I_3+231I_3 \rightarrow 72=1319I_3 \rightarrow$$

$$I_3=\frac{72}{1319}=0.05459\text{ A}\approx\boxed{0.055\text{ A}} \ ; \ I_2=\frac{6+34I_3}{11}=0.714\text{ A}\approx\boxed{0.71\text{ A}} \ ; \ I_1=I_2+I_3=\boxed{0.77\text{ A}}$$